Monads

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Agenda

- Recap of Functors
- Recap of Applicative
- Monads

Functor[1][2]

```haskell
class Functor f where
    fmap :: (a -> b) -> f a -> f b
    (<$) :: a -> f b -> f a
```

Functors Laws

- Must preserve identity

```haskell
fmap id = id
```

- Must preserve composition of morphism

```haskell
fmap (f . g)  ==  fmap f . fmap g
```

[1] Category Design Pattern
[2] Functor Design Pattern

Higher order kinds[3]

- For something to be a functor, it has to be a first order kind.

[3] Haskell's Kind System

Applicative

```
class Functor f => Applicative (f :: TYPE -> TYPE) where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

(<$>) :: Functor f =>       (a -> b) -> f a -> f b
(<*>) :: Applicative f => f (a -> b) -> f a -> f b

fmap f x = pure f <*> x
```

Examples

```
pure (+1) <*> [1..3]
[2, 3, 4]

[(*2), (*3)] <*> [4, 5]
[8,10,12,15]

("Woo", (+1)) <*> (" Hoo!", 0)
("Woo Hoo!", 1)

(Sum 2, (+1)) <*> (Sum 0, 0)
(Sum {getSum = 2}, 1)

(Product 3, (+9)) <*> (Product 2, 8)
(Product {getProduct = 6}, 17)

(,) <$> [1, 2] <*> [3, 4]
[(1,3),(1,4),(2,3),(2,4)]
```

Use cases[4]

```haskell
Person
  <$> parseString "name" o
  <*> parseInt "age" o
  <*> parseTelephone "telephone" o
```

Can also be written as

```haskell
liftA3 Person
  (parseString "name" o)
  (parseInt "age" o)
  (parseTelephone "telephone" o)
```

[4] FP Complete - Crash course to Applicative syntax

Use cases[5]

```haskell
parsePerson :: Parser Person
parsePerson = do
  string "Name: "
  name <- takeWhile (/= 'n')
  endOfLine
  string "Age: "
  age <- decimal
  endOfLine
  pure $ Person name age
```

[5] FP Complete - Crash course to Applicative syntax

Use cases[6]

```
helper :: () -> Text -> () -> () -> Int -> () -> Person
helper () name () () age () = Person name age

parsePerson :: Parser Person
parsePerson = helper
  <$> string "Name: "
  <*> takeWhile (/= 'n')
  <*> endOfLine
  <*> string "Age: "
  <*> decimal
  <*> endOfLine
```

[6] FP Complete - Crash course to Applicative syntax

Lifting

- Seeing Functor as unary lifting and Applicative as n-ary lifting

```
liftA0 :: Applicative f => (a)                     -> (f a)
liftA1 :: Functor     f => (a -> b)                -> (f a -> f b)
liftA2 :: Applicative f => (a -> b -> c)           -> (f a -> f b -> f c)
liftA3 :: Applicative f => (a -> b -> c -> d) -> (f a -> f b -> f c -> f d)
liftA4 :: Applicative f => ..
```

Where `liftA0 = pure` and `liftA1 = fmap`.

Monoidal functors

- Remember Monoid?

```
class Monoid m where
  mempty :: m
  mappend :: m -> m -> m

($)   ::   (a -> b) ->   a ->   b
(<$>) ::   (a -> b) -> f a -> f b
(<*>) :: f (a -> b) -> f a -> f b


mappend ::     f          f      f
($) ::       (a -> b) ->   a ->   b
<*> ::     f (a -> b) -> f a -> f b


instance Monoid a => Applicative ((,) a) where
  pure x = (mempty, x)
  (u, f) <*> (v, x) = (u `mappend` v, f x)
```

Where are monoids again

```
fmap (+1) ("blah", 0)
("blah",1)

("Woo", (+1)) <*> (" Hoo!", 0)
("Woo Hoo!", 1)

(,) <$> [1, 2] <*> [3, 4]
[(1,3),(1,4),(2,3),(2,4)]

liftA2 (,) [1, 2] [3, 4]
[(1,3),(1,4),(2,3),(2,4)]
```

Function apply

- Applying a function to an `effectful` argument

```
(<$>) :: Functor m     =>   (a -> b)   -> m a -> m b
(<*>) :: Applicative m => m (a -> b)   -> m a -> m b
(=<<) :: Monad m       =>   (a -> m b) -> m a -> m b
```

Applicative laws

```haskell
-- Identity
pure id <*> v = v

-- Composition
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)

-- Homomorphism
pure f <*> pure x = pure (f x)

-- Interchange
u <*> pure y = pure ($ y) <*> u
```

Operators[7]

- `pure` wraps up a pure value into some kind of Applicative
- `liftA2` applies a pure function to the values inside two `Applicative` wrapped values
- `<$>` operator version of `fmap`
- `<*>` apply a wrapped function to a wrapped value
- `*>`, `<*`

[7] FP Complete - Crash course to Applicative syntax

Monad, is that you?[8]

[8] The Unreasonable Effectiveness of Metaphor

Motivation - I

```haskell
safeInverse :: Float -> Maybe Float
safeInverse 0 = Nothing
safeInverse x = Just (1 / x)

safeSqrt :: Float -> Maybe Float
safeSqrt x = case x <= 0 of
  True -> Nothing
  False -> Just (sqrt x)

sqrtInverse1 :: Float -> Maybe (Maybe Float)
sqrtInverse1 x = safeInverse <$> (safeSqrt x)
```

Motivation - I

```haskell
joinMaybe :: Maybe (Maybe a) -> Maybe a
joinMaybe (Just x) = x
joinMaybe Nothing = Nothing

sqrtInverse2 :: Float -> Maybe Float
sqrtInverse2 x = joinMaybe $ safeInverse <$> (safeSqrt x)

-- In general
-- join :: Monad m => m (m a) -> m a
```

Motivation - II

```haskell
(>>=) :: Maybe a -> (a -> Maybe b) -> Maybe b
x >>= f = case x of
  (Just x') -> f x'
  Nothing -> Nothing

sqrtInverse :: Float -> Maybe Float
sqrtInverse x = (>>=) (safeSqrt x) safeInverse

-- >>= is also known as `bind`

-- In general
-- (>>=) :: Monad m => m a -> (a -> m b) -> m b
```

Motivation - III

```
(>=>) :: (a -> Maybe b) -> (b -> Maybe c) -> (a -> Maybe c)
f >=> g = \x -> case f x of
  Just x -> g x
  Nothing -> Nothing

sqrtInverse3 :: Float -> Maybe Float
sqrtInverse3 = safeSqrt >=> safeInverse

-- In general
-- (>=>) :: Monad m => (a -> m b) -> (b -> m c) -> (a -> m c)
```

Motivations

- Flattening
- Sequencing
- Composition

Monad

```haskell
class Applicative m => Monad (m :: Type -> Type) where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b

import Control.Monad (join)

join :: Monad m => m (m a) -> m a
```

do notation

```haskell
main :: IO ()
main = do
  putStrLn "What is your name?"
  name <- getLine
  let greeting = "Hello, " ++ name
  putStrLn greeting
```

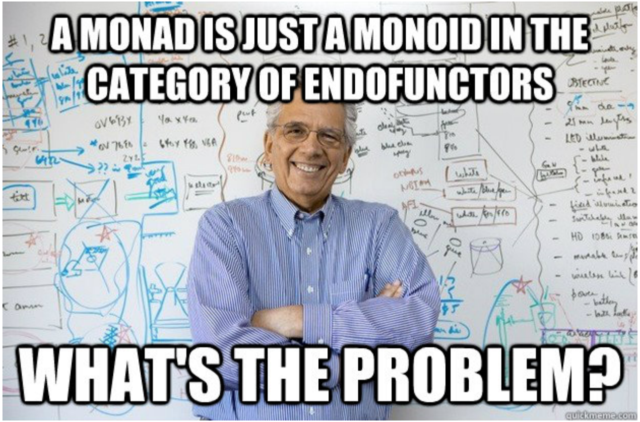
Monad laws

```haskell
-- Left identity
return x >>= f == f x

-- Right identity
x >>= return == x

-- Associativity
m >>= (\x -> k x >>= h) == (m >>= k) >>= h
```

???


A MONAD IS JUST A MONOID IN THE CATEGORY OF ENDOFUNCTORS
WHAT'S THE PROBLEM?
quickmeme.com

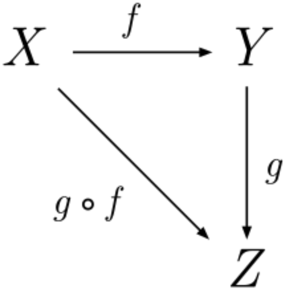
Monoids recap

```haskell
class Semigroup m where
  (<>) :: m -> m -> m

class Semigroup m => Monoid m where
  mempty :: m

  -- defining mappend is unnecessary, it copies from Semigroup
  mappend :: m -> m -> m
  mappend = (<>)
```

Some Math



- Category: a set of objects and arrows
- Arrows between objects (morphisms): functions mapping one object to another
- Two categories: **Set** and **Hask**

Categories

- **Set**
  - Category of sets
  - Every arrow, function from one set to another
- **Hask**
  - Similar to **Set**
  - Objects are Haskell types like `Int` instead of `Z` or `R`
  - Arrows between objects `a` & `b` are functions of type `a -> b`
  - `a -> b` also a `Type` in **Hask**
  - If `A -> B` and `B -> C`, then `A -> C` $\sim=$ `.` in **Hask**
  - Fun fact: Function composition forms a monoid! (See Endo).

Monads are monoids. . .

In Haskell

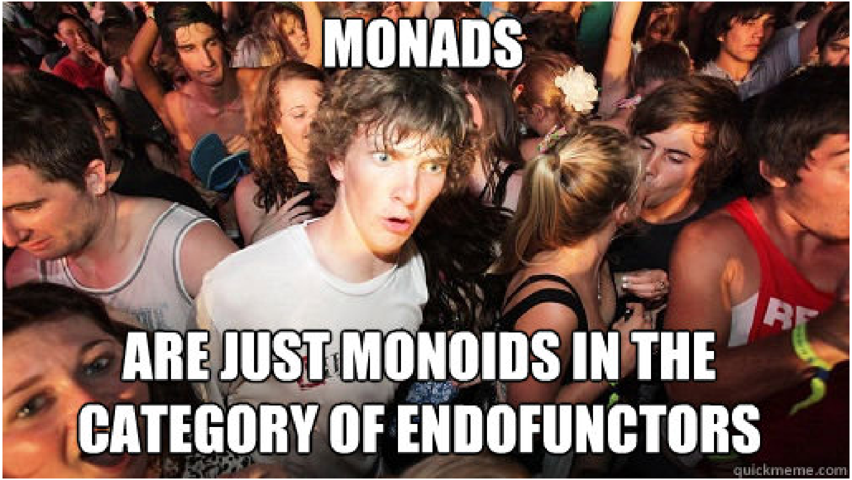
- Only work with **Hask**, so functors all map back to **Hask**.
- Functor typeclass are a special type of functor called **endofunctors**
- **endofunctors** map a category back to itself
- Monad is a monoid where

```haskell
-- Operation
>==>

-- Identity
return

-- Set
Type
a -> m b
```

Now?


MONADS
ARE JUST MONOIDS IN THE
CATEGORY OF ENDOFUNCTORS
quickmeme.com

Contrasts with Monad

- No data dependency between `f a` and `f b`
- Result of `f a` can't possibly influence the behaviour of `f b`
- That needs something like `a -> f b`

Applicative vs Monads

- Applicative
  - Effects
  - Batching and aggregation
  - Concurrency/Independent
    - Parsing context free grammar
    - Exploring all branches of computation (see `Alternative`)
- Monads
  - Effects
  - Composition
  - Sequence/Dependent
    - Parsing context sensitive grammar
    - Branching on previous results

Weaker but better

- Weaker than monads but thus also more common
- Lends itself to optimisation (See Facebook's Haxl project)
- Always opt for the least powerful mechanism to get things done
- No dependency issues or branching? just use applicative

State monad

```haskell
newtype State s a = State { runState :: s -> (a, s) }

instance Functor (State s) where
  fmap :: (a -> b) -> State s a -> State s b
  fmap f (State sa) = State $ \s -> let (a, s) = sa s in (f a, s)

instance Applicative (State s) where
  pure :: a -> State s a
  pure a = State $ \s -> (a, s)

  (<*>) :: State s (a -> b) -> State s a -> State s b
  State f <*> State g = State $ \s -> let (aTob, s') = f s in
                                      let (a, s'') = g s' in
                                          (aTob a, s'')
```

State monad

```haskell
instance Monad (State s) where
  return = pure
  (>>=) :: State s a
        -> (a -> State s b)
        -> State s b
  (State f) >>= g = State $ \s -> let (a, s') = f s
                                      ms = runState $ g a
                                  in ms s'
  (>>) :: State s a
       -> State s b
       -> State s b
  State f >> State g = State $ \s -> let (_, s') = f s
                                     in g s'

get :: State s s
get = State $ \s -> (s, s)
```

State monad

```
put :: s -> State s ()
put s = State $ \_ -> ((), s)

modify :: (s -> s) -> State s ()
modify f = get >>= \x -> put (f x)

eval :: State s a -> s -> a
eval (State sa) x = let (a, _) = sa x
                    in a
```

Context

```haskell
type Stack = [Int]

empty :: Stack
empty = []

pop :: State Stack Int
pop = State $ \(x:xs) -> (x, xs)

push :: Int -> State Stack ()
push a = State $ \xs -> ((), a:xs)

tos :: State Stack Int
tos = State $ \(x:xs) -> (x, x:xs)
```

Context

```
stackManip :: State Stack Int
stackManip = do
    push 10
    push 20
    a <- pop
    b <- pop
    push (a+b)
    tos

testState = eval stackManip empty
```

Reader monad

```
class Monad m => MonadReader r m | m -> r where
  ask :: m r
  local :: (r -> r) -> m a -> m a
```

Context

```haskell
import Control.Monad.Reader

tom :: Reader String String
tom = do
    env <- ask
    return (env ++ " This is Tom.")

jerry :: Reader String String
jerry = do
  env <- ask
  return (env ++ " This is Jerry.")
```

Context

```haskell
tomAndJerry :: Reader String String
tomAndJerry = do
    t <- tom
    j <- jerry
    return (t ++ " " ++ j)

runJerryRun :: String
runJerryRun = runReader tomAndJerry "Who is this?"
```

Questions

- Reach out on
  - Email: me@sanchayanmaity.net
  - Mastodon: sanchayanmaity.com
  - Telegram: t.me/SanchayanMaity
  - Blog: sanchayanmaity.net